Mecheleciv

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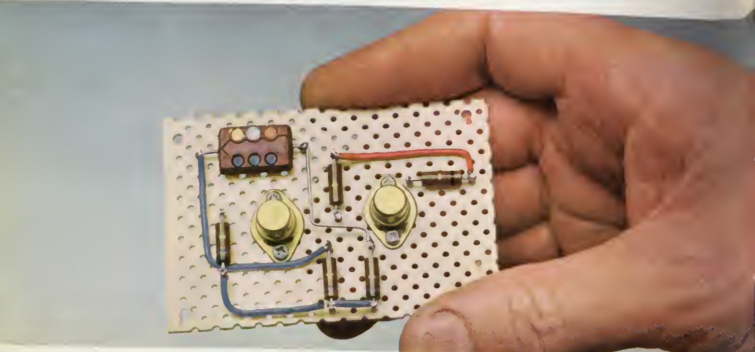
THE GEORGE WASHINGTON UNIVERSITY

APRIL 1965

This tiny device is an ultra-reliable Westinghouse TV amplifier



It can replace all these wires and electronic parts

"With fewer parts and connections," Westinghouse scientists decided, "we'd have a much more reliable amplifier."

So they developed a new kind, using techniques called "molecular electronics." The device is a functional electronic block, one quarter-inch square. (That's the capsule. Actually, the electronics are on a bit of specially-treated silicon the size of a chip diamond.)

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You can be sure if it's Westinghouse

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Mecheleciv

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ARTICLES

page

DEPARTMENTS

COVER

On the cover this month, our artist has depicted the flow field over a set of turbine blades, since two of our articles this month are concerned with the analysis of fluid flow. Our Indian is back with us again.

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In last month's editorial, we introduced the idea of a "playboy engineer." There we attempted to reform the traditional "playboy" by giving him a good solid job (professional engineering). We combined the positive traits of a fun-loving playboy and a hard-working engineer in order to achieve a balanced, satisfying life. This month we will examine the playboy engineer's modern, practical principles of right and wrong, his "ethical philosophy" (if you'll pardon that liberal arts expression).

Why does the playboy engineer concern himself with a subject so far removed from technology? He does so for two reasons. First, the playboy engineer, you remember, believes that life can never be all engineering and no play.

PRINCIPLE OF UTILITY:	Every course of action has its consequences. The course of action is right if the consequences tend to aid in the pursuit of a full, satisfying life.

By this principle, wasting needed study time is wrong for the student who wants good grades. Wearing a topless bathing suit would be wrong for a girl if she were trampled in the resulting riot. It would be wrong for an engineer to be shy around girls if he wanted to be a playboy engineer. Notice that by this principle, actions that are wrong for some people can be right for others. Notice also that an action having bad consequences will be wrong irregardless of the intent behind the action. The person with good

The PLAYBOY ENGINEER'S *Philosophy*

Secondly, he knows that without standards of behavior his life can easily become "dominated by obsolete tradition, by impersonal social forces, or by unconscious mental drives." So the playboy engineer forms practical ethical principles to guide his life. He wants to know just "how far to go" tonight with his girl, how much (if any) of last year's lab report he should copy, or how much (if any) substandard materials should he use in building a stadium.

Usually when it comes to morals, we are followers rather than leaders. We follow the advice and example of people around us -- parents, friends, police, and occasionally even magazine editors. The playboy engineer certainly doesn't see anything basically wrong with this, but he feels that it never hurts to think things through for himself. Even if he ends up agreeing with others, which he usually does, he at least agrees on the basis of rationality, rather than authority or conformity. Sometimes, however, in the full spirit of freedom, he disagrees.

Because the playboy engineer is trained in the application of scientific principles, he would like very much to apply this same engineering method in his approach to ethics. He wants some basic, general principles which can be easily remembered and easily applied to practical moral problems. In this editorial, we will suggest two such principles. They are not new; they can be found in almost any philosophy book. Nor are they perfect; they may not apply in every situation. Nevertheless, the playboy engineer finds them satisfactory at least as a first approximation.

The moral principles which the playboy engineer has adopted are equally applicable to both work and play. You may remember that the playboy engineer works and plays "with all his might." In this respect he sees no difference between his work and his play; they are both a part of living, and he lives life to the hilt. So he sees no need for a different set of ethics for work and play. What then are the principles which the playboy engineer applies to his whole life?

intentions may not be held responsible for the action, but the action itself will be wrong in any case.

This first principle is basic both to the utilitarian philosophy of John Stuart Mill and the Hedonistic philosophy of Epicurus. The main criticism against it is that it is not always possible to foresee every consequence of an action. This situation, however, is certainly not new to the playboy engineer who must very often make engineering design decisions based on incomplete knowledge. Thus he has adopted the principle of utility, in spite of the fact that it does not always give clear-cut answers, because it provides a practical and objective basis for his playboy engineering ethics.

The principle of utility must not be interpreted in such a way as to make an action right even though it hurts another's chances for a full and satisfying life. To guard against this possibility, the playboy engineer adds a second principle to the first.

PRINCIPLE OF SOCIAL RESPONSIBILITY:	Man is a social animal. He does not live independently of others; but rather his life is inter-related and interdependent on the lives of others.

This principle means that the playboy engineer cannot attain a full and satisfying life alone. The social characteristic of the playboy engineer was clearly demonstrated, surprisingly enough, by Charles Darwin in the fourth chapter of his Descent of Man. Even on the basis of his own engineering experience, the playboy engineer recognizes the advantage of group effort. He realizes that engineers working separately often lack the knowledge, skill, or power to achieve their objectives. Furthermore, he knows that our present standard of living would be impossible without social organization — the division and specialization of labor. In our complex society,

the playboy engineer must depend on many others for his own well-being.

Application of the Principles To Work:

When it comes to ethics of work, the playboy engineer is fortunate that his profession in 1947 adopted the "Canons of Ethics for Engineers."[c] These ethics are primarily an application of the principle of social responsibility. Because society does not have the necessary technical knowledge, it must place complete trust in the engineer. So any engineer who acts in such a way as to damage that trust is hurting the entire profession (and thus himself). This same principle applies to the playboy engineer while he is still in school. Since copying another's lab report usually means that the engineering student will be a less capable engineer after graduation, such action is wrong.

It is often said that scientists and engineers are not really responsible for the sociological problems which result from their efforts. The playboy engineer regards such statements as obvious violations of the principle of social responsibility. For who is responsible? Society, some may answer. But are not scientists and engineers a part of society? The playboy engineer willingly accepts the responsibility for such things as environmental pollution, over-crowded highways, and the destructiveness of modern warfare. The playboy engineer has also done his share in creating the problem of over-population.

Application of the Principles To Play:

Of the many possible applications of our ethical principles to leisure time activities, we will concentrate on only one -- sex. Quite frankly, this is one case where our principles have not provided the playboy engineer with a simple yes or no answer. And yet, they have told him something. For example, the old-fashioned negative and prohibitive attitude towards sex is definitely wrong. And certainly an obsession with sex, that gives it an importance all out of proportion, has unhappy consequences that make it just as wrong. But what about the practical problem of deciding "how far to go" with a girl on a date? The consequences of "going all the way" are not so serious today thanks to such "technological developments" as penicillin and oral contraceptives. But does this mean that it is now perfectly alright to treat sex as another form of entertainment? The playboy engineer is not sure; in fact, he suspects that there is something more to human sexuality than just having fun and babies. At any rate, this is one time that the playboy engineer is going to have to do his own thinking instead of reading editorials. —J.L.E.

THE NEW CURRICULUM – A SUBJECTIVE EVALUATION

Press Relations Committee

Sigma Tau Fraternity

An invigorating discussion at the Open Forum on the "new" curriculum gave students and faculty considerable insight into problem areas, misconceptions, and the general opinion of the curriculum. By and large, the students seemed to like the program, but felt the comprehensive exams were not really fair. Also, it was brought to light that the counseling program needs a drastic overhaul -- both on the part of the student and on the part of the faculty.

Poor Counseling

Although it was agreed that the new program is good, poor counseling was hit very hard. There are not any pre-requisites listed in the catalog, and few advisors know the pre-requisites for all courses. This leaves students enrolled in courses for which they are grossly unprepared; or what is more common and worse, the student is only partially prepared and consequently spends all semester running like the devil to catch up. Also, it was brought to light that numerous students and unfortunately a few faculty members do not take advising seriously. To sum up this aspect of the discussion, advisors and especially new faculty members should be better indoctrinated in the program and in the school, pre-requisite material for every course should be published, and advising should be taken more seriously.

Adaptability and Comprehensives

Dean Grisamore referred to this as the "nth" curriculum rather than the new curriculum. The curriculum must change every year or the school will not keep pace with technological advances. Under the new curriculum, new courses may be added at will, and new degrees or fields of specialization may be added relatively easily. For example, a degree in space engineering may readily be added. But this is only an administrative advantage of the new program. The real advantage lies in individual student flexibility. The essential discipline of self teaching is inherently encouraged. Similarly, if Prof. A is teaching course B and I don't like Prof. A, said one person, I can simply learn the material from a book or two.

The faculty wanted to emphasize that the comprehensive exams are a guidance to the student to tell the student whether or not he has the knowledge he will need when he graduates. Unless you are a complete flop, there is no such thing as flunking the exams. You simply learn the material in areas where the exam indicates you are weak.

Thus this new program gives you complete leeway to learn material which is of interest to you and at your level, but it provides check and evaluation to tell you whether or not you are going to be able to reach 1st base when you begin practicing engineering.

VECTOR EQUATIONS IN FLUID MECHANICS

by Fred Flatow

This paper discusses some of the applications of vector equations to fluid mechanics. Some of the more important equations are developed. They are compared to similar equations in electro-magnetic theory where appropriate.

Introduction

The use of vectors in mechanics, hydrodynamics and electrodynamics simplifies and condenses the exposition and helps to make mathematical and physical concepts more tangible.

In general, the velocity of a particle is represented by a vector. Fluid mechanics and hydrodynamics deal with the problem of analyzing the motion of a fluid filling space. In this case, the velocities of different particles are in general independent of one another, and every point has its own special vector, representing its velocity. The moving, continuous fluid represents a vector field.

One can speak of the field of a physical quantity, when the quantity is considered from the point of view of its dependence upon position in a region of space. The field may be either a scalar field or a vector field. In general, its values are assumed to be continuous, with the exception of a finite number of surfaces, lines and points.

In general, it is found that in irrotational motion the electric field $\vec{E}$ takes the place of the velocity field $\vec{V}$. For the analysis of vortex flows of inviscid fluids, the Biot - Savart law may be used. Here the magnetic intensity $\vec{H}$ becomes the velocity $\vec{V}$, and the current density $\vec{J}$ becomes the vorticity $\vec{\omega}$.

Point Sources

This development is specifically designed to show the similarities between the velocity $\vec{V}$ of a fluid, and the electric field $\vec{E}$.

The fluid is assumed to be incompressible and it is assumed to have a density of 1. Point sources and sinks are postulated.

The mass of a fluid which per second crosses a surface of a sphere of radius r, with its center at the source, is equal to the strength e of the source

$$e = \iint \vec{V} \cdot d\vec{S} = 4\pi r^2 V_n$$

from which

$$\vec{V} = \frac{e}{4\pi r^2}\vec{a}_r$$

Also, the velocity potential is defined such that the velocity $\vec{V}$ is considered to be the negative gradient of the velocity potential, or

$$\vec{V} = -\nabla\phi$$

Similarly, the electric field $\vec{E}$ from a point charge Q is

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2}\vec{a}_r = -\nabla\xi$$

where ξ is the electrical potential.

For both the fluid and electrical cases, the vector fields are irrotational, since each can be derived as the gradient of a scalar field.

Furthermore, for linear media, the principle of superposition holds and for the velocity field

$$\vec{V} = \sum_{i=1}^{n}\vec{V}_i = -\nabla\phi$$

where

$$\phi = \sum_{i=1}^{n}\frac{e_i}{4\pi r_i}$$

For the electric field

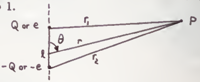
$$\vec{E} = \sum_{i=1}^{n}\vec{E}_i = -\nabla\xi$$

where

$$\xi = \sum_{i=1}^{n}\frac{Q_i}{4\pi\epsilon r_i}$$

As an example of the above, the potential of a fluid doublet will be calculated and compared to the potential from an electric dipole.

If l is the distance from source to sink (+e to -e) there results a system of moment m = el. The analysis is valid for distances large compared to l.



At a point P, by superposition, the total potential is

$$\phi = \phi_1 + \phi_2 = \frac{m}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$r_1 \simeq r - \frac{l}{2}\cos\theta \quad ; \quad r_2 = r + \frac{l}{2}\cos\theta$$

Substituting this into the equation for potential above, there results

$$\phi = \frac{m\cos\theta}{4\pi r^2}$$

Similarly, for the electric dipole a potential expression is obtained such that

$$\xi = \frac{Ql\cos\theta}{4\pi\epsilon r^2} = \frac{k\cos\theta}{4\pi r^2} \quad \text{where } k = \frac{Ql}{\epsilon}$$

Equation of Continuity

Applying the principle of conservation of mass, a mathematical equation is developed, which states that the net efflux rate of mass through a closed surface equals the rate of decrease inside the volume surrounded by the surface.

Let ρ be the density of a fluid; $\vec{V}$ its velocity and P a point (x,y,z). ρ and $\vec{V}$ are functions of x,y,z and t.

In a fixed closed surface S, the mass is increasing at a rate

$$\frac{\partial}{\partial t}\int \rho\, dV = \int \frac{\partial \rho}{\partial t}\, dV$$

This rate equals the rate at which fluid is entering S

$$-\oint \bar{n}\cdot \rho\vec{V}\, dS$$

Equating these two equations

$$\int \frac{\partial \rho}{\partial t}\, dV = -\int \vec{n}\cdot \rho\vec{V}\, dS = -\int \nabla\cdot(\rho\vec{V})\, dV$$

by the use of the divergence theorem.

From this, the equation of continuity given below is obtained:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$$

For an incompressible fluid, the density remains constant in time and space and

$$\nabla\cdot\vec{V} = 0$$

For electro-magnetic theory, a similar law of continuity can be developed.

Let $\vec{J}$ be current density, ρ charge density. Then, the net current flow into a volume is accompanied by an increase in charge within the volume.

$$-\oint_s \vec{J}\cdot d\vec{S} = \frac{\partial}{\partial t}\int_v \rho\, dV$$

where the left hand side gives the net inflow of current (coulombs per second) and the right hand side represents the net rate of increase in charge (coulombs per second). By the use of the divergence theorem, this becomes

$$\nabla\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

This can also be written as

$$\nabla\cdot\sigma\vec{E} = -\frac{\partial \rho}{\partial t}$$

For steady currents

$$\frac{\partial \rho}{\partial t} = 0$$

and

$$\nabla\cdot\vec{J} = \nabla\cdot\vec{E} = 0$$

These equations equal those developed above for fluid mechanics.

Euler's Equation

The Eulerian or statistical method of treating fluid motion intends to find the velocity $\overline{V}$, density and pressure p of a fluid as a function of time t at a point P. Starting with Newton's Law,

$$d\vec{F} = \frac{d}{dt}\, d(m\vec{V}) \quad \text{where } \vec{V} = \vec{V}(x, y, z, t)$$

$$d\vec{F} = dm\left(V_x\frac{\partial\vec{V}}{\partial x} + V_y\frac{\partial\vec{V}}{\partial y} + V_z\frac{\partial\vec{V}}{\partial z} + \frac{\partial\vec{V}}{\partial t}\right)$$

$$d\vec{F} = dm\left[\vec{V}\cdot\nabla + \frac{\partial}{\partial t}\right]\vec{V}$$

Assume that the only forces present are surface forces and gravity.

Surface force = $-(\nabla p)\, dv$

Gravity = $-g\rho dv\,\vec{k} = g(\nabla z)\rho dv$

Dividing through by $\rho dv = dm$ the following is obtained:

$$-\frac{1}{\rho}\nabla p - g(\nabla z) = (\vec{V}\cdot\nabla)\vec{V} + \frac{\partial\vec{V}}{\partial t}$$

This is Euler's equation.

(DON'T PANIC THERE'S MORE)

LaPlace's Equation

Another Helmholtz theorem states that the most general vector field can be derived from the negative gradient of a scalar potential ϕ and the curl of a vector potential $\vec{A}$. Hence, the velocity $\vec{V}$ of a fluid is written as

$$\vec{V} = -\nabla\phi - \nabla\times\vec{A}$$

For the general irrotational motion of an inviscid fluid, (or potential flow)

$$\vec{V} = \nabla\phi$$

By Kelvin's Circulation Theorem, if the vorticity of a fluid vanishes at any instant, it will remain zero thereafter.

Using the equation of continuity

$$\nabla\cdot\vec{V} = \frac{1}{\rho}\frac{d\vec{V}}{dt}$$

substitute the potential expression for $\overline{V}$ into it and obtain

$$\nabla^2\phi = \frac{d}{dt}(\ln\rho)$$

For incompressible fluid, the right hand term is zero, and

$$\nabla^2\phi = 0$$

This is LaPlace's equation. It is satisfied by the velocity potential of an irrotational and incompressible fluid.

Similarly, for electro-magnetic theory the electric field

$$\vec{E} = -\nabla\xi - \frac{\partial\vec{A}}{\partial t}$$

For the static case

$$\vec{E} = -\nabla\xi$$

From Maxwell's equation

$$\epsilon\nabla\cdot\vec{E} = \rho$$

Substituting the potential expression for $\overline{E}$, one obtains Poisson's equation:

$$\epsilon\nabla^2\xi = -\rho$$

For a charge-free region, the right hand term is zero and LaPlace's equation results:

$$\nabla^2\xi = 0$$

Therefore, in a source-free region, the electric field, expressed as the negative of the gradient of an electric scalar potential, obeys LaPlace's equation.

--Continued on page 15

FACULTY SPOTLIGHT

While most of the engineering students at George Washington University have seen Dean Smith many times and know who he is, very few of us are aware of his impressive background.

He first came to George Washington University in 1958 as a Professorial lecturer in Engineering Administration. He served in that position for two years when he left to teach math at the University of Maryland. In 1961 he returned to George Washington as Director of the Engineering Administration Curriculum, and one year later became Assistant Dean (Academic) of the School of Engineering.

Besides teaching at George Washington and Maryland Universities, he was an instructor in Surveying, School of Technology, College of the City of New York; a Lecturer (Planning), Rensselaer Polytechnic Institute; and a Lecturer at the Naval War College.

Dean Smith's college education consists of a Bachelor of Science in Physics and Civil Engineering in 1930, and a Bachelor of Civil Engineering in 1932. These two degrees were offered in a five year course at the College of the City of New York. He then attended New York University where he received his M.S. in 1936, and his Ph.D. in 1940; both in Civil Engineering and in Physics.

While working for his advanced degrees, he was employed by the City of New York as an Engineer. In the period 1941 to 1950 he served in the Navy in such capacities as Executive officer and Commanding officer of Naval Construction Battalion and later of Naval Construction Regiment, and as Public Works Officer. Dean Smith became Deputy Director of the Atlantic Division of the Bureau of Yards and Docks where he served for five years. From 1956 to 1958 he was the principle Engineering advisor for the Commander of Naval forces in the Far East.

In his job as Assistant Dean (Academic), Dean Smith affects our education as much or more than anybody in the University. His specific duties are:

a) Taking necessary administrative actions in any matters directly relating to instruction;

Assistant Dean Herbert E. Smith

b) Maintaining academic liaison with other instructional elements in the University;
c) Maintaining liaison with student affairs;
d) Recommending to the Dean and Dean's Council for approval and action:

   1) Curricula, courses, and course offerings;
   2) Teaching assignments of instructors;
   3) Instruction needs (faculty, facilities, equipment);
   4) Student disciplinary actions for academic reasons;
   5) Academic standards and regulations.

With regards to teaching, Dean Smith feels that there should be a great deal of student participation in class. He also feels that students should express a deeper interest in the humanities.

His professional societies include the American Society of Civil Engineers, Society of American Military Engineers, U.S. Naval Institute, and the American Management Association.

He has done extensive public speaking on a variety of topics, and has written numerous articles in his field. He is proficient in French, and has traveled in the Far East, Africa, Europe, and South America.

It is good to know that a person with Dean Smith's character and background has such an effect in determining the course of our education.

LAPLACE TRANSFORMS

One of the more elegant approaches to the solution of differentional equations is that utilizing a transformation — the changing of the functions involved into functions of a new variable. One of the most useful of these transformations is the Laplace transform, which is used extensively in circuit analysis. Such an approach, of course, need not be confined to electrical engineering; it is equally valid for the solution of differential equations regardless of their physical significance.

The Laplace transform involves the transformation of a function, $f(t)$, of one variable t into a function, $F(s)$, of the variable s according to the definition:

$$F(s) = \int_0^\infty f(t)\epsilon^{-st}dt;\ t > 0$$

This transform has very useful properties. Consider the transform of the first derivative of $y(t)$:

$$Y'(s) = \int_0^\infty y'(t)\epsilon^{-st}dt.$$

Integrating by parts yields:

$$Y'(s) = \lim_{t\to\infty} \epsilon^{-st}\, y(t) - y(0) + s\cdot\int_0^\infty \epsilon^{-st} y(t)\,dt$$
$$= -y(0) + sY(s),$$

the limit term being zero for all practical problems.

Likewise, it can be shown that:

$$Y''(s) = \int_0^\infty y''(t)\epsilon^{-st}dt =$$
$$-y'(0) - sy(0) + s^2Y(s),$$

and a similar pattern is followed for higher derivatives.

This transformation results in a simplification of differential equations. Consider the following second-order equation with constant coefficients:

$$ay''(t) + by'(t) + cy(t) = f(t)$$

Performing the Laplace transformation on each function yields:

$$a[-y^1(0) - sy(0) + s^2Y(s)]$$
$$+ b[-y(0) + sY(s)] + cY(c) = F(s)$$

or:

$$Y(s) = \frac{1}{as^2 + bs + c}$$
$$[F(s) + ay^1(0) + (as + b)y(0)]$$

The desired solution, $y(t)$, can now be obtained by inverse transformation.

The advantages should now be apparent. First, the problem in differential calculus has been reduced to one in simple algebra. Second, the thorny problem of applying initial conditions to a general solution, as in the classical method, has been eliminated; initial conditions are included directly in the Laplace transform. This technique is applicable for any constant-coefficient equation.

The technique has been greatly simplified by the tabulation of the Laplace transforms of simple functions in many common reference sources. This eliminates any direct integration in making the transformation from the t-domain to the s-domain and the inverse transformation back to the t-domain. The latter is a particularly messy problem.

The most serious difficulty encountered in using Laplace transforms is algebraic — the reduction of the resultant quotient of polynomials in s which constitutes $Y(s)$ into a form which is recognizable for the use of tables. This can be best accomplished by the use of partial fractions, breaking down the quotient of polynomials into a sum of simpler terms. Such a procedure will almost always yield terms which can be individually transformed back to the t-domain by direct reference to tables.

Consider the application of this procedure to the solution of a particular integral:

$$y''(t) + 4y'(t) + 8y(t) = 1$$

with $y'(0) = 1$ and $y(0) = 0$. Making the transformation into the s-domain:

$$4[-y(0) + sY(s)] + 8Y(s)$$
$$-y'(0) - sy(0) + s^2Y(s) = \frac{1}{s}$$

where 1/s is the Laplace transform of 1.

Inserting the numerical values for $y'(o)$ and $y(o)$ yields:

$$-1 + s^2Y(s) + 4sY(s) + 8Y(s) = \frac{1}{s}$$

solving for $Y(s)$: $$Y(s) = \frac{1+s}{s(s^2+4s+8)}.$$

Splitting this quotient into partial fractions:

$$Y(s) = \frac{A}{S} + \frac{B}{s^2+4s+8} + \frac{Cs}{s^2+4s+8}$$

$$= \frac{A(s^2+4s+8) + Bs + Cs^2}{s(s^2+4s+8)} = \frac{1+s}{s(s^2+4s+8)}$$

Equating coefficients of powers of s:

$$8A = 1,\ 4A + B = 1,\ A + C = 0$$

which implies:

$$A = ⅛,\ B = ½,\ C = -⅛$$

and:

$$Y(s) = ⅛s + ½\ \frac{1}{s^2+4s+8} - ⅛\ \frac{S}{s^2+4s+8}$$

$$= ⅛\left[\frac{1}{s} + 3\,\frac{2}{(s+2)^2+4} - \frac{2+s}{(s+2)^2+4}\right]$$

This expression can be transformed back into the t-domain by reference to a table which indicates tha

$\frac{1}{s}$ transforms to 1

$\frac{2}{(s+2)^2+4}$ transforms to ϵ^{-2t}Sin2t

$\frac{s+2}{(s+2)^2+4}$ transforms to ϵ^{-2t}Cos2t

Therefore the desired solution $y(t)$ is:

$$y(t) = ⅛[1 + \epsilon^{-2t}(3\text{Sin}2t - \text{Cos}2t)].$$

Needless to say, the preceding brief discussion does not represent the entire picture of Laplace transforms. The many theorems and mechanical subtleties which make the application of this theory much simpler could not possibly be included here. The underlying mathematical principles have not been discussed. These topics are thoroughly treated in many standard texts. But even the simple procedures outlined here represent a powerful method for attacking many otherwise complex problems.

$F(t)$	$f(s)$
$F(t)$	$\int_0^\infty e^{-st}F(t)\,dt$
$AF(t) + BG(t)$	$Af(s) + Bg(s)$
$F'(t)$	$sf(s) - F(+0)$
$F^{(n)}(t)$	$s^n f(s) - s^{n-1}F(+0) - s^{n-2}F'(+0) - \cdots - F^{(n-1)}(+0)$
$\int_0^t F(\tau)\,d\tau$	$\frac{1}{s}f(s)$
$\int_0^t \int_0^\tau F(\lambda)\,d\lambda\,d\tau$	$\frac{1}{s^2}f(s)$
$\int_0^t F_1(t-\tau)F_2(\tau)\,d\tau = F_1 * F_2$	$f_1(s)f_2(s)$
$tF(t)$	$-f'(s)$
$t^nF(t)$	$(-1)^n f^{(n)}(s)$
$\frac{1}{t}F(t)$	$\int_s^\infty f(x)\,dx$
$e^{at}F(t)$	$f(s-a)$
$F(t-b)$, where $F(t) = 0$ when $t < 0$	$e^{-bs}f(s)$
$\frac{1}{c}F\left(\frac{t}{c}\right)$ $(c > 0)$	$f(cs)$
$\frac{1}{c}e^{\frac{bt}{c}}F\left(\frac{t}{c}\right)$ $(c > 0)$	$f(cs - b)$
$F(t)$, when $F(t+a) = F(t)$	$\frac{\int_0^a e^{-st}F(t)\,dt}{1 - e^{-as}}$
$F(t)$, when $F(t+a) = -F(t)$	$\frac{\int_0^a e^{-st}F(t)\,dt}{1 + e^{-as}}$
$F_1(t)$, the half-wave rectification of $F(t)$	$\frac{f(s)}{1 - e^{-as}}$
$F_2(t)$, the full-wave rectification of $F(t)$	$f(s)\coth\frac{as}{2}$
$\sum_1^m \frac{p(a_n)}{q'(a_n)}e^{a_nt}$	$\frac{p(s)}{q(s)}$, $q(s) = (s-a_1)(s-a_2)\cdots(s-a_m)$

$f(s)$	$F(t)$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$ $(n = 1, 2, \ldots)$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
$s^{-\frac{3}{2}}$	$2\sqrt{\frac{t}{\pi}}$
$s^{-(n+\frac{1}{2})}$ $(n = 1, 2, \ldots)$	$\frac{2^n t^{n-\frac{1}{2}}}{1 \times 3 \times 5 \cdots (2n-1)\sqrt{\pi}}$
$\frac{\Gamma(k)}{s^k}$ $(k > 0)$	t^{k-1}
$\frac{1}{s-a}$	e^{at}
$\frac{1}{(s-a)^2}$	te^{at}
$\frac{1}{(s-a)^n}$ $(n = 1, 2, \ldots)$	$\frac{1}{(n-1)!}t^{n-1}e^{at}$
$\frac{\Gamma(k)}{(s-a)^k}$ $(k > 0)$	$t^{k-1}e^{at}$
$\frac{1}{(s-a)(s-b)}$	$\frac{1}{a-b}(e^{at} - e^{bt})$

$f(s)$	$F(t)$
$\dfrac{1}{s \sinh ks}$	$F(t) = 2(n-1)$ when $(2n-3)k < t < (2n-1)k$ $(t > 0)$
$\dfrac{1}{s \cosh ks}$	$M(2k, t+3k) + 1 = 1 + (-1)^n$ when $(2n-3)k < t < (2n-1)k$ $(t > 0)$
$\dfrac{1}{s} \coth ks$	$F(t) = 2n - 1$ when $2k(n-1) < t < 2kn$
$\dfrac{k}{s^2+k^2} \coth \dfrac{\pi s}{2k}$	$\lvert \sin kt \rvert$
$\dfrac{1}{(s^2+1)(1-e^{-\pi s})}$	$\begin{cases} \sin t \text{ when } (2n-2)\pi < t < (2n-1)\pi \\ 0 \quad \text{when } (2n-1)\pi < t < 2n\pi \end{cases}$
$\dfrac{1}{s} e^{-(k/s)}$	$J_0(2\sqrt{kt})$
$\dfrac{1}{\sqrt{s}} e^{-(k/s)}$	$\dfrac{1}{\sqrt{\pi t}} \cos 2\sqrt{kt}$
$\dfrac{1}{\sqrt{s}} e^{k/s}$	$\dfrac{1}{\sqrt{\pi t}} \cosh 2\sqrt{kt}$
$\dfrac{1}{s^{\frac{3}{2}}} e^{-(k/s)}$	$\dfrac{1}{\sqrt{\pi k}} \sin 2\sqrt{kt}$
$\dfrac{1}{s^{\frac{3}{2}}} e^{k/s}$	$\dfrac{1}{\sqrt{\pi k}} \sinh 2\sqrt{kt}$
$\dfrac{1}{s^\mu} e^{-(k/s)}$ $(\mu > 0)$	$\left(\dfrac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$
$\dfrac{1}{s^\mu} e^{k/s}$ $(\mu > 0)$	$\left(\dfrac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$
$e^{-k\sqrt{s}}$ $(k > 0)$	$\dfrac{k}{2\sqrt{\pi t^3}} \exp\left(-\dfrac{k^2}{4t}\right)$
$\dfrac{1}{s} e^{-k\sqrt{s}}$ $(k \geqq 0)$	$\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
$\dfrac{1}{\sqrt{s}} e^{-k\sqrt{s}}$ $(k \geqq 0)$	$\dfrac{1}{\sqrt{\pi t}} \exp\left(-\dfrac{k^2}{4t}\right)$
$s^{-\frac{3}{2}} e^{-k\sqrt{s}}$ $(k \geqq 0)$	$2\sqrt{\dfrac{t}{\pi}} \exp\left(-\dfrac{k^2}{4t}\right) - k \operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
$\dfrac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})}$ $(k \geqq 0)$	$-e^{ak}e^{a^2t} \operatorname{erfc}\left(a\sqrt{t} + \dfrac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
$\dfrac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})}$ $(k \geqq 0)$	$e^{ak}e^{a^2t} \operatorname{erfc}\left(a\sqrt{t} + \dfrac{k}{2\sqrt{t}}\right)$
$\dfrac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ e^{-\frac{1}{2}at} I_0(\frac{1}{2}a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$
$\dfrac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ J_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$
$\dfrac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ I_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$
$\dfrac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}}$ $(k \geqq 0)$	$J_0(a\sqrt{t^2+2kt})$
$e^{-ks} - e^{-k\sqrt{s^2+a^2}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \dfrac{ak}{\sqrt{t^2-k^2}} J_1(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$
$e^{-k\sqrt{s^2-a^2}} - e^{-ks}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \dfrac{ak}{\sqrt{t^2-k^2}} I_1(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$

$f(s)$	$F(t)$
$\dfrac{a^\nu e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}\,(\sqrt{s^2+a^2}+s)^\nu}$ $(\nu > -1)$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \left(\dfrac{t-k}{t+k}\right)^{\frac{1}{2}\nu} J_\nu(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$
$\dfrac{1}{s} \log s$	$\Gamma'(1) - \log t$ $[\Gamma'(1) = -0.5772]$
$\dfrac{1}{s^k} \log s$ $(k > 0)$	$t^{k-1} \left\{ \dfrac{\Gamma'(k)}{[\Gamma(k)]^2} - \dfrac{\log t}{\Gamma(k)} \right\}$
$\dfrac{\log s}{s-a}$ $(a > 0)$	$e^{at}[\log a - \operatorname{Ei}(-at)]$
$\dfrac{\log s}{s^2+1}$	$\cos t \operatorname{Si} t - \sin t \operatorname{Ci} t$
$\dfrac{s \log s}{s^2+1}$	$-\sin t \operatorname{Si} t - \cos t \operatorname{Ci} t$
$\dfrac{1}{s} \log(1+ks)$ $(k > 0)$	$-\operatorname{Ei}\left(-\dfrac{t}{k}\right)$
$\log \dfrac{s-a}{s-b}$	$\dfrac{1}{t}(e^{bt} - e^{at})$
$\dfrac{1}{s} \log(1+k^2s^2)$	$-2 \operatorname{Ci}\left(\dfrac{t}{k}\right)$
$\dfrac{1}{s} \log(s^2+a^2)$ $(a > 0)$	$2 \log a - 2 \operatorname{Ci}(at)$
$\dfrac{1}{s^2} \log(s^2+a^2)$ $(a > 0)$	$\dfrac{2}{a}[at \log a + \sin at - at \operatorname{Ci}(at)]$
$\log \dfrac{s^2+a^2}{s^2}$	$\dfrac{2}{t}(1 - \cos at)$
$\log \dfrac{s^2-a^2}{s^2}$	$\dfrac{2}{t}(1 - \cosh at)$
$\arctan \dfrac{k}{s}$	$\dfrac{1}{t} \sin kt$
$\dfrac{1}{s} \arctan \dfrac{k}{s}$	$\operatorname{Si}(kt)$
$e^{k^2s^2} \operatorname{erfc}(ks)$ $(k > 0)$	$\dfrac{1}{k\sqrt{\pi}} \exp\left(-\dfrac{t^2}{4k^2}\right)$
$\dfrac{1}{s} e^{k^2s^2} \operatorname{erfc}(ks)$ $(k > 0)$	$\operatorname{erf}\left(\dfrac{t}{2k}\right)$
$e^{ks} \operatorname{erfc} \sqrt{ks}$ $(k > 0)$	$\dfrac{\sqrt{k}}{\pi\sqrt{t}(t+k)}$
$\dfrac{1}{\sqrt{s}} \operatorname{erfc}(\sqrt{ks})$	$\begin{cases} 0 & \text{when } 0 < t < k \\ (\pi t)^{-\frac{1}{2}} & \text{when } t > k \end{cases}$
$\dfrac{1}{\sqrt{s}} e^{ks} \operatorname{erfc}(\sqrt{ks})$ $(k > 0)$	$\dfrac{1}{\sqrt{\pi(t+k)}}$
$\operatorname{erf}\left(\dfrac{k}{\sqrt{s}}\right)$	$\dfrac{1}{\pi t} \sin(2k\sqrt{t})$
$\dfrac{1}{\sqrt{s}} e^{k^2/s} \operatorname{erfc}\left(\dfrac{k}{\sqrt{s}}\right)$	$\dfrac{1}{\sqrt{\pi t}} e^{-2k\sqrt{t}}$
$K_0(ks)$	$\begin{cases} 0 & \text{when } 0 < t < k \\ (t^2-k^2)^{-\frac{1}{2}} & \text{when } t > k \end{cases}$
$K_0(k\sqrt{s})$	$\dfrac{1}{2t} \exp\left(-\dfrac{k^2}{4t}\right)$
$\dfrac{1}{s} e^{ks} K_1(ks)$	$\dfrac{1}{k} \sqrt{t(t+2k)}$
$\dfrac{1}{\sqrt{s}} K_1(k\sqrt{s})$	$\dfrac{1}{k} \exp\left(-\dfrac{k^2}{4t}\right)$
$\dfrac{1}{\sqrt{s}} e^{k/s} K_0\left(\dfrac{k}{s}\right)$	$\dfrac{2}{\sqrt{\pi t}} K_0(2\sqrt{2kt})$
$\pi e^{-ks} I_0(ks)$	$\begin{cases} [t(2k-t)]^{-\frac{1}{2}} & \text{when } 0 < t < 2k \\ 0 & \text{when } t > 2k \end{cases}$
$e^{-ks} I_1(ks)$	$\begin{cases} \dfrac{k-t}{\pi k \sqrt{t(2k-t)}} & \text{when } 0 < t < 2k \\ 0 & \text{when } t > 2k \end{cases}$
$-e^{as} \operatorname{Ei}(-as)$	$\dfrac{1}{t+a}$ $(a > 0)$

$f(s)$	$F(t)$
$\dfrac{s}{(s-a)(s-b)}$	$\dfrac{1}{a-b}(ae^{at}-be^{bt})$
$\dfrac{1}{(s-a)(s-b)(s-c)}$	$-\dfrac{(b-c)e^{at}+(c-a)e^{bt}+(a-b)e^{ct}}{(a-b)(b-c)(c-a)}$
$\dfrac{1}{s^2+a^2}$	$\dfrac{1}{a}\sin at$
$\dfrac{s}{s^2+a^2}$	$\cos at$
$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\sinh at$
$\dfrac{s}{s^2-a^2}$	$\cosh at$
$\dfrac{1}{s(s^2+a^2)}$	$\dfrac{1}{a^2}(1-\cos at)$
$\dfrac{1}{s^2(s^2+a^2)}$	$\dfrac{1}{a^3}(at-\sin at)$
$\dfrac{1}{(s^2+a^2)^2}$	$\dfrac{1}{2a^3}(\sin at-at\cos at)$
$\dfrac{s}{(s^2+a^2)^2}$	$\dfrac{t}{2a}\sin at$
$\dfrac{s^2}{(s^2+a^2)^2}$	$\dfrac{1}{2a}(\sin at+at\cos at)$
$\dfrac{s^2-a^2}{(s^2+a^2)^2}$	$t\cos at$
$\dfrac{s}{(s^2+a^2)(s^2+b^2)}\ (a^2\neq b^2)$	$\dfrac{\cos at-\cos bt}{b^2-a^2}$
$\dfrac{1}{(s-a)^2+b^2}$	$\dfrac{1}{b}e^{at}\sin bt$
$\dfrac{s-a}{(s-a)^2+b^2}$	$e^{at}\cos bt$
$\dfrac{3a^2}{s^3+a^3}$	$e^{-at}-e^{at/2}\left(\cos\dfrac{at\sqrt{3}}{2}-\sqrt{3}\sin\dfrac{at\sqrt{3}}{2}\right)$
$\dfrac{4a^3}{s^4+4a^4}$	$\sin at\cosh at-\cos at\sinh at$
$\dfrac{s}{s^4+4a^4}$	$\dfrac{1}{2a^2}\sin at\sinh at$
$\dfrac{1}{s^4-a^4}$	$\dfrac{1}{2a^3}(\sinh at-\sin at)$
$\dfrac{s}{s^4-a^4}$	$\dfrac{1}{2a^2}(\cosh at-\cos at)$
$\dfrac{8a^3s^2}{(s^2+a^2)^3}$	$(1+a^2t^2)\sin at-at\cos at$
$\dfrac{1}{s}\left(\dfrac{s-1}{s}\right)^n$	$L_n(t)=\dfrac{e^t}{n!}\dfrac{d^n}{dt^n}(t^ne^{-t})$
$\dfrac{s}{(s-a)^{3/2}}$	$\dfrac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
$\sqrt{s-a}-\sqrt{s-b}$	$\dfrac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
$\dfrac{1}{\sqrt{s}+a}$	$\dfrac{1}{\sqrt{\pi t}}-ae^{a^2t}\operatorname{erfc}(a\sqrt{t})$
$\dfrac{\sqrt{s}}{s-a^2}$	$\dfrac{1}{\sqrt{\pi t}}+ae^{a^2t}\operatorname{erf}(a\sqrt{t})$
$\dfrac{\sqrt{s}}{s+a^2}$	$\dfrac{1}{\sqrt{\pi t}}-\dfrac{2a}{\sqrt{\pi}}e^{-a^2t}\int_0^{a\sqrt{t}}e^{\lambda^2}\,d\lambda$
$\dfrac{1}{\sqrt{s}(s-a^2)}$	$\dfrac{1}{a}e^{a^2t}\operatorname{erf}(a\sqrt{t})$
$\dfrac{1}{\sqrt{s}(s+a^2)}$	$\dfrac{2}{a\sqrt{\pi}}e^{-a^2t}\int_0^{a\sqrt{t}}e^{\lambda^2}\,d\lambda$
$\dfrac{b^2-a^2}{(s-a^2)(b+\sqrt{s})}$	$e^{a^2t}[b-a\operatorname{erf}(a\sqrt{t})]-be^{b^2t}\operatorname{erfc}(b\sqrt{t})$
$\dfrac{1}{\sqrt{s}(\sqrt{s}+a)}$	$e^{a^2t}\operatorname{erfc}(a\sqrt{t})$

$f(s)$	$F(t)$
$\dfrac{1}{(s+a)\sqrt{s+b}}$	$\dfrac{1}{\sqrt{b-a}}e^{-at}\operatorname{erf}(\sqrt{b-a}\sqrt{t})$
$\dfrac{b^2-a^2}{\sqrt{s}(s-a^2)(\sqrt{s}+b)}$	$e^{a^2t}\left[\dfrac{b}{a}\operatorname{erf}(a\sqrt{t})-1\right]+e^{b^2t}\operatorname{erfc}(b\sqrt{t})$
$\dfrac{(1-s)^n}{s^{n+\frac{1}{2}}}$	$\dfrac{n!}{(2n)!\sqrt{\pi t}}H_{2n}(\sqrt{t})$
$\dfrac{(1-s)^n}{s^{n+\frac{3}{2}}}$	$-\dfrac{n!}{\sqrt{\pi}(2n+1)!}H_{2n+1}(\sqrt{t})$
$\dfrac{\sqrt{s+2a}}{\sqrt{s}}-1$	$ae^{-at}[I_1(at)+I_0(at)]$
$\dfrac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-\frac{1}{2}(a+b)t}I_0\left(\dfrac{a-b}{2}t\right)$
$\dfrac{\Gamma(k)}{(s+a)^k(s+b)^k}\ (k>0)$	$\sqrt{\pi}\left(\dfrac{t}{a-b}\right)^{k-\frac{1}{2}}e^{-\frac{1}{2}(a+b)t}\times I_{k-\frac{1}{2}}\left(\dfrac{a-b}{2}t\right)$
$\dfrac{1}{(s+a)^{\frac{1}{2}}(s+b)^{\frac{3}{2}}}$	$te^{-\frac{1}{2}(a+b)t}\left[I_0\left(\dfrac{a-b}{2}t\right)+I_1\left(\dfrac{a-b}{2}t\right)\right]$
$\dfrac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s+2a}+\sqrt{s}}$	$\dfrac{1}{t}e^{-at}I_1(at)$
$\dfrac{(a-b)^k}{(\sqrt{s+a}+\sqrt{s+b})^{2k}}\ (k>0)$	$\dfrac{k}{t}e^{-\frac{1}{2}(a+b)t}I_k\left(\dfrac{a-b}{2}t\right)$
$\dfrac{(\sqrt{s+a}+\sqrt{s})^{-2\nu}}{\sqrt{s}\sqrt{s+a}}\ (\nu>-1)$	$\dfrac{1}{a^\nu}e^{-\frac{1}{2}at}I_\nu\left(\dfrac{1}{2}at\right)$
$\dfrac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
$\dfrac{(\sqrt{s^2+a^2}-s)^\nu}{\sqrt{s^2+a^2}}\ (\nu>-1)$	$a^\nu J_\nu(at)$
$\dfrac{1}{(s^2+a^2)^k}\ (k>0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-\frac{1}{2}}J_{k-\frac{1}{2}}(at)$
$(\sqrt{s^2+a^2}-s)^k\ (k>0)$	$\dfrac{ka^k}{t}J_k(at)$
$\dfrac{(s-\sqrt{s^2-a^2})^\nu}{\sqrt{s^2-a^2}}\ (\nu>-1)$	$a^\nu I_\nu(at)$
$\dfrac{1}{(s^2-a^2)^k}\ (k>0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-\frac{1}{2}}I_{k-\frac{1}{2}}(at)$
$\dfrac{e^{-ks}}{s}$	$S_k(t)=\begin{cases}0 \text{ when } 0<t<k\\ 1 \text{ when } t>k\end{cases}$
$\dfrac{e^{-ks}}{s^2}$	$\begin{cases}0 & \text{when } 0<t<k\\ t-k & \text{when } t>k\end{cases}$
$\dfrac{e^{-ks}}{s^\mu}\ (\mu>0)$	$\begin{cases}0 & \text{when } 0<t<k\\ \dfrac{(t-k)^{\mu-1}}{\Gamma(\mu)} & \text{when } t>k\end{cases}$
$\dfrac{1-e^{-ks}}{s}$	$\begin{cases}1 \text{ when } 0<t<k\\ 0 \text{ when } t>k\end{cases}$
$\dfrac{1}{s(1-e^{-ks})}=\dfrac{1+\coth\frac{1}{2}ks}{2s}$	$1+[t/k]=n$ when $(n-1)k<t<nk$ $(n=1,2,\ldots)$
$\dfrac{1}{s(e^{ks}-a)}$	$\begin{cases}0 \text{ when } 0<t<k\\ 1+a+a^2+\cdots+a^{n-1}\\ \quad\text{when } nk<t<(n+1)k\end{cases}$ $(n=1,2,\ldots)$
$\dfrac{1}{s}\tanh ks$	$M(2k,t)=(-1)^{n-1}$ when $2k(n-1)<t<2kn$ $(n=1,2,\ldots)$
$\dfrac{1}{s(1+e^{-ks})}$	$\dfrac{1}{2}M(k,t)+\dfrac{1}{2}=\dfrac{1-(-1)^n}{2}$ when $(n-1)k<t<nk$
$\dfrac{1}{s^2}\tanh ks$	$H(2k,t)$

MECH MISS . . .

Our lovely Engineers' Queen, Miss Candace Lee Scherer, is Theta Tau's Sweetheart. These pages attest as to why the fraternity and other engineers think she is the prettiest girl in school. This petite (she says she is 5' 2" "in her shoes") 20 year old has lived in the South Pacific and in many countries in Western Europe while her father was a naval officer. This young lady's well chosen mickname, Candy, indicates why her personality and charm has won the hearts of several young men.

If you are wondering about three vital statistics, the sum is thirty inches more than her height. If you think about it a little and also know that the upper exceeds the lower by one and the smallest is the number whose reciprocal times a thousand is 45.4545.

Candy is the second Delta Gamma in as many years to be the Engineers' Queen. If there are more like her in her sorority, let us make it three years in a row.

Pictures by William Burry

FLOW FORMATION IN A SUDDENLY ACCELERATED RECTANGULAR CONTAINER

by Douglas L. Jones

ABSTRACT

The Navier-Stokes equation for laminar flow in the x direction was solved for a suddenly accelerated rectangular container. The container was open at each end and it was accelerated to a constant velocity, U_0, instantaneously at initial time.

The flow formation was then given by the solution of the Navier-Stokes equation with time as a parameter. The flow was given as a dimensionless velocity profile ($\frac{U}{U_0}$) as a function of $\frac{y}{l}$ and $\frac{z}{l}$, the lateral dimensionless distances. Curves of the velocity profile for several times were then plotted and from them constant velocity lines were drawn as a second set of graphical results.

INTRODUCTION

The purpose of this paper was to indicate the solution of the Navier-Stokes equation for flow formation in parallel flow. The configuration involved was that of sudden acceleration of a rectangular, open-end container in the direction of the axis of symmetry (Figure 1). At initial time the velocity was instantly increased from zero to U_0 and held there. Interaction of inertia and viscous forces caused the fluid velocity to gradually increase to the same velocity as the container. It was desired to determine the velocity profiles at various specified times so that the flow formation could be analyzed.

The fluid was assumed incompressible with constant viscosity. The flow was laminar which allowed the analysis of parallel flow to be applied to this case. Flow of the type described would allow no pressure gradient in any direction so the gradient in the x direction was assumed zero.

Under the assumptions above the Navier-Stokes equations reduce to the single equation:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\nu}\frac{\partial u}{\partial t}$$

where u is the flow in the x direction and ν is the kinematic viscosity. This equation is similar to an equation expressing unsteady, two-dimensional heat flow. The equation is known as Fourier's equation and its solution is indicated in several books on Heat Transfer. Since no solution of Fourier's equation for fluid flow could be found, the solution for heat transfer was adapted to fluid flow.

The solution obtained was a product of four infinite series. Suitable boundary and initial conditions were utilized to evaluate the constants of integration. The velocity profile was then determined and expressed in the form of a dimensionless number.

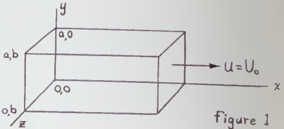


Figure 1

ANALYSIS

For parallel flow the Navier-Stokes equation is given in the form

$$\rho\frac{\partial u}{\partial t} = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

where ρ is the constant density and μ is the constant viscosity. Since no pressure gradient exists in this problem the gradient dp/dt was assumed zero and we are left with the equation:

$$\rho\frac{\partial u}{\partial t} = \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad \text{or} \quad \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\nu}\frac{\partial u}{\partial t}$$

where v is the kinematic viscosity.

The equation above is analogous to Fourier's Equation for heat flow and we can solve it by using methods applicable to the solution of Fourier's Equation. To allow greater ease of evaluating the constants in the solution we will define $U = u - U_0$ and substitute it into the equation above. We get the following equation

$$\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = \frac{1}{\nu}\frac{\partial U}{\partial t}$$

with the boundary conditions at t = 0.

$$U = 0 \text{ for } y = 0, a$$
$$U = 0 \text{ for } z = 0, b.$$

We will now proceed to solve the equation by separation of variables.

Assume U = Y(y,t) Z(z,t) and substitute into the equation to get

$$Z\frac{\partial^2 Y}{\partial y^2} + Y\frac{\partial^2 Z}{\partial z^2} = \frac{1}{\nu}Y\frac{\partial Z}{\partial t} + \frac{1}{\nu}Z\frac{\partial Y}{\partial t}$$

Dividing through by YZ we have the result:

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \frac{1}{\nu Z}\frac{\partial Z}{\partial t} + \frac{1}{\nu Y}\frac{\partial Y}{\partial t}$$

We now equate terms in Y and Z to get

$$\frac{\partial^2 Y}{\partial y^2} = \frac{1}{\nu}\frac{\partial Y}{\partial t}$$

Separating Y(y,t) into functions of y and t, we get

$$Y = (A\sin\gamma y + B\cos\gamma y)\,e^{-\nu\gamma^2 t}$$

where $(-\gamma^2)$ is the constant of separation and A and B are arbitrary constants. Also by separating

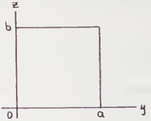
Z(z,t) into functions of z and t, we obtain

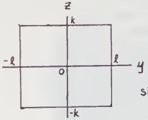
$$Z = \left(C \sin \beta z + D \cos \beta z\right) e^{-\nu \beta^2 t}$$

where $(-\beta^2)$ is the constant of separation. Thus the general solution is

$$U = \left[\left(A \sin \gamma y + B \cos \gamma y\right) e^{-\gamma^2 \nu t}\right]\left[\left(C \sin \beta z + D \cos \beta z\right) e^{-\nu \beta^2 t}\right]$$

We find it more convenient to express the distribution with respect to the center rather than to a corner.





$$a = 2l, \; b = 2k$$

$$\sin \frac{(2n+1)\pi y}{a} = \cos \frac{(2n+1)\pi y}{2l}$$

The final solution, in terms of dimensionless velocity profiles, is:

$$\frac{U}{U_0} = 1 - \frac{16}{\pi^2} \sum_{m=0}^{\infty} \frac{(-1)^m \cos \frac{(2m+1)\pi y}{2l} e^{-\frac{\nu (2m+1)^2 \pi^2 t}{4l^2}}}{2m+1} \sum_{n=0}^{\infty} \frac{(-1)^n \cos \frac{(2n+1)\pi z}{2k} e^{-\frac{\nu (2n+1)^2 \pi^2 t}{4k^2}}}{2n+1}$$

For ease of obtaining numerical results we will let l = k and concern ourselves with a square container. We will plot the dimensionless velocity profiles as functions of y/l which will be the same as z/l. We will assume a value for $\nu = 10^{-5}$ which is that of water at 75° F. and let y = 0 to find the value along the z/l coordinate line. The profiles will also be determined along the diagonal where y/l = z/l and these will give sufficient information to obtain the desired results.

BIBLIOGRAPHY

Carslaw, H. S. and J. C. Jaeger. Conductors of Heat in Solids. Oxford: Clarendon Press, 1947.

Colarco, J. and D. L. Jones. "Two Dimensional Transient Heat Conduction." Unpublished paper.

Ingersoll, L. R. and O. J. Fobel. Mathematical Theory of Heat. Boston: Ginn & Co., 1913.

Schlicting, H. Boundary Layer Theory. New York: McGraw-Hill Book Co., Inc., 1962.

Schneider, P. J. Conduction Heat Transfer. Addison-Wesley Publishing Co., Cambridge, Mass., 1955.

Shames, I. H. Mechanics of Fluids. New York: McGraw-Hill Book Co., Inc., 1962.

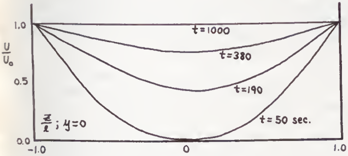


Figure 2. Velocity Profile Along Axes

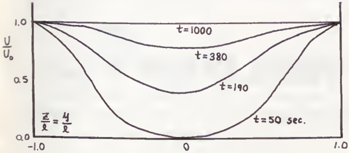


Figure 3. Velocity Profile along Diagonal

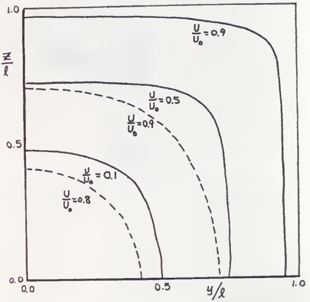


Figure 4. Constant Velocity Curves

—— t = 50 sec.

- - - - t = 380 sec.

STUDENT GOVERNMENT

by Bob Mullen

Hallelujah! The Student Council has finally removed the self-imposed albatross which has burdened it since last fall. The Council voted to accomplish the necessary reforms in the Constitution through amendments rather than struggle further with the impossible task of composing a new Constitution which two-thirds of the Council would agree on. The most pressing area demanding reform is the representation to non-resident students and dorm-dwellers without flooding the Council with sixteen more members as was proposed previously. All members of the Council recognize the need for commuter representation and a compromise will be worked out in committee and submitted to the Council.

The most important legislation passed thus far was an amendment which struck out the clause forbidding groups with national affiliation from being recognized on campus. (Greeks, Young Republicans, and Young Democrats were exceptions.) While this may mean that students can be approached by organizations which are not in the mainstream of University thought, it is my opinion that the presence of a few "extremist" groups on campus differentiates between a college which is just a box of "ticky-tacky" turning out naive stereotypes of a college graduate and a college which gives its students at least some insight into what is going on in the world. We hope the Student Life Committee will approve this amendment.

Congratulations to the new Engineer's Council members:

Introductory:	Stacy Deming Burton Goldstein
Intermediate:	Chip Young Orv Standifer
Advanced:	John Starke Huda Faruki

The Engineer's Council will elect its officers for the coming year. at its next meeting. Anyone desiring to work in any capacity for the Council or for Mecheleciv should make themselves known to myself, a member of the Council, or a member of the Mecheleciv staff. The positions will be filled within the next month so now is the time to apply.

VECTOR EQUATIONS—Continued from page 7

Bernoulli's Equation

This equation stipulates that the sum of pressure energy (flow work) per unit mass, potential energy of position per unit mass and kinetic energy per unit mass is conserved along two points. When there is frictionless, incompressible, steady and irrotational flow, it may be applied between any two points.

This equation is developed from Euler's equation. When there is steady flow, the latter is written as

$$-\frac{\nabla P}{\rho} - g\nabla z = (\vec{V}\cdot\nabla)\vec{V}$$

By transforming the right hand side of the equation by vector identities, one obtains

$$-\frac{\nabla P}{\rho} - g\nabla z = \nabla\left(\frac{V^2}{2}\right) - \vec{V}\times\nabla\times\vec{V}$$

For irrotational flow

$$\frac{\nabla P}{\rho} + g\nabla z + \nabla\left(\frac{V^2}{2}\right) = 0$$

Taking the dot product of each term with the displacement vector $d\vec{r}$, the following results

$$\frac{dP}{\rho} + g\,dz + \frac{dV^2}{2} = 0$$

since

$$\nabla z\cdot d\vec{r} = dz\ ;\ \nabla P\cdot d\vec{r} = dp\ ;\left(\nabla\frac{V^2}{2}\right)\cdot d\vec{r} = d\left(\frac{V^2}{2}\right)$$

Limiting this equation to an incompressible flow, taking g as a constant and integrating one obtains Bernoulli's equation, which is expressed as

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{constant}$$

REFERENCES

Abraham, Max, and Becker, Richard, The Classical Theory of Electricity and Magnetism, Hafner Publishing Company, Inc., New York, N. Y., 1949.

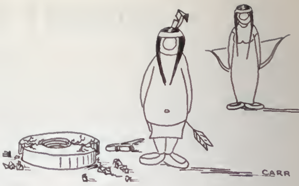
Brand, Louis, Vector Analysis, John Wiley & Sons, Inc., New York, N. Y., 1957.

Plonsey, R., and Collins, R. E., Principles and Applications of Electromagnetic Fields, McGraw-Hill Book Company, Inc., New York, N. Y., 1961.

Sommerfeld, Arnold, Mechanics of Deformable Bodies, Academic Press Inc., New York, N. Y., 1950.

Shames, I. H., Mechanics of Fluids, McGraw-Hill Book Company, Inc., New York, N. Y., 1962.

THE SHAFT



The young couple drove away on their honeymoon blissfully unaware of the sign their friends had put on the rear bumper: AMATEUR NIGHT.

* * *

M.E.: "I'm grasping for words."
Coed: "I think you are looking in the wrong place."

* * *

One of the prettier girls I know always says, "To err is human but it feels divine."

* * *

"Why can't you measure the position and momentum of a particle exactly?" asked Heisenberg uncertainly.

* * *

GIRL: "Isn't that a lovely moon tonight?"
BOY: "I'm not interested in astronomy now, and besides I'm in no position to say."

* * *

SHE: It's funny, you go out just as much as I do, yet your laundry bill is always double mine.
HER: Yes, but I go with an engineer.

* * *

Engineer's Motto: Keep frowning -- and get credit for thinking.

A bra manufacturer who sells his product under the slogan "Every Girl Wants EMBARGO" was asked why he picked "EMBARGO" for a trade name. "At first glance you might think it's foolish," he said, "but spelled backwards, it has tremendous sales appeal."

* * *

It is usually easy to hold a pretty girl tight. It's getting her tight that is sometimes a problem.

* * *

Overheard in Tompkins Hall:
"Is she frigid? She thinks sex is just a German number."

* * *

A college professor calling the roll:
"Robinson?"
"Here."
"George Smith?"
"Here."
"Mary Smith?"
"Here."
"Wanamaker?"
Chorus: "Yes!"

* * *

If a gal wants to wear slacks, she'd better make sure that the end justifies the jeans.

* * *

Strip poker is a game that begins according to Hoyle and ends according to Kinsey.

An inspector asked a new M.E. the purpose of a bolt with a left-handed thread and got this bewildering reply:
"A bolt with a left-handed thread is a bolt which the tighter it's screwed the looser it gets."

* * *

A guy called his girl one night and asked "Got anything on for tonight?"
"Yes," replied the chick, "and it's staying on."

* * *

Doctors may bury their mistakes but most men end up supporting theirs.

* * *

The president of a large corporation has come up with a perfect way to end staff conferences. He merely says, "All those opposed to my plan say I resign!"

* * *

The visitor at a nudist camp was intrigued by a nudist who had a beard that flowed all the way to his knees. "How come the beard?" asked the tourist.
"Well," said the bearded one, "somebody has to go out for coffee."

* * *

It's easier for a girl to walk the straight and narrow if she happens to be built that way.

INDEX TO ADVERTISERS

This is one of our mechanical engineers making a mistake

They are to wed in June, and the guy had better shut up before she gets miffed. A gal has every right to resent the implication that the betrothed outpoints her in understanding of sewing and fabrics and what's good or bad about them. Even if it's true. Which it is. We have made him a pro at it.

It is our crafty intent to stop at nothing in our efforts to make garments or fabric furnishings that carry our identification tag (as for KODEL Fiber) so pleasing to the ultimate buyer in every way that she will attribute the satisfaction all to the fiber and look for that tag evermore. This means we put mechanical engineers, chemical engineers, chemists and—yes—physicists to work freshening up the technology of dyeing, knitting, weaving, sewing, and the other elderly arts practiced not by us but by our customers' customers.

As in all the other industries in which we participate and for which we seek scientific and engineering recruits — photography, information retrieval, aerospace, plastics, graphic arts, x-ray, chemicals—there is much to challenge the intellectually ambitious in satisfying the common yearnings of mankind for adornment of the person and the home. Past technical accomplishments in fibers and fabrics, weak by comparison with what can be anticipated when fresh, better informed minds pitch in, have sufficed nonetheless to create the present affluence where there is plenty of money on hand to do what smart people will tell us to do. All we need are more smart people.

Drop us a line. From polymer theory to workable yarn and from workable yarn to clothes on the back, rugs on the floor, and curtains on the windows extends a long row of assorted disciplines and aptitudes.

EASTMAN KODAK COMPANY,

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Kodak

Should You Work for a Big Company?

An interview with General Electric's S. W. Corbin, Vice President and General Manager, Industrial Sales Division.

S. W. CORBIN

■ Wells Corbin heads what is probably the world's largest industrial sales organization, employing more than 8000 persons and selling hundreds of thousands of diverse products. He joined General Electric in 1930 as a student engineer after graduation from Union College with a BSEE. After moving through several assignments in industrial engineering and sales management, he assumed his present position in 1960. He was elected a General Electric vice president in 1963.

Q. Mr. Corbin, why should I work for a big company? Are there some special advantages?

A. Just for a minute, consider what the scope of product mix often found in a big company means to you. A broad range of products and services gives you a variety of starting places now. It widens tremendously your opportunity for growth. Engineers and scientists at General Electric research, design, manufacture and sell thousands of products from microminiature electronic components and computer-controlled steel-mill systems for industry; to the world's largest turbine-generators for utilities; to radios, TV sets and appliances for consumers; to satellites and other complex systems for aerospace and defense.

Q. How about attaining positions of responsibility?

A. How much responsibility do you want? If you'd like to contribute to the design of tomorrow's atomic reactors—or work on the installation of complex industrial systems—or take part in supervising the manufacture of exotic machine-tool controls—or design new hardware or software for G-E computers—or direct a million dollars in annual sales through distributors—you can do it, in a big company like General Electric, if you show you have the ability. There's no limit to responsibility . . . except your own talent and desire.

Q. Can big companies offer advantages in training and career development programs?

A. Yes. We employ large numbers of people each year so we can often set up specialized training programs that are hard to duplicate elsewhere. Our Technical Marketing Program, for example, has specialized assignments both for initial training and career development that vary depending on whether you want a future in sales, application engineering or installation and service engineering. In the Manufacturing Program, assignments are given in manufacturing engineering, factory supervision, quality control, materials management or plant engineering. Other specialized programs exist, like the Product Engineering Program for you prospective creative design engineers, and the highly selective Research Training Program.

Q. Doesn't that mean there will be more competition for the top jobs?

A. You'll always find competition for a good job, no matter where you go! But in a company like G.E. where there are 150 product operations, with broad research and sales organizations to back them up, you'll have less chance for your ambition to be stalemated. Why? Simply because there are more top jobs to compete for.

Q. How can a big company help me fight technological obsolescence?

A. Wherever you are in General Electric, you'll be helping create a rapid pace of product development to serve highly competitive markets. As a member of the G-E team, you'll be on the leading edge of the wave of advancement—by adapting new research findings to product designs, by keeping your customers informed of new product developments that can improve or even revolutionize their operations, and by developing new machines, processes and methods to manufacture these new products. And there will be classwork too. There's too much to be done to let you get out of date!

FOR MORE INFORMATION on careers for engineers and scientists at General Electric, write Personalized Career Planning, General Electric, Section 699-12, Schenectady, N. Y. 12305

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